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Minimum variance portfolio: the art of constraints

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Abstract

Minimum variance portfolio is an attractive theoretical opportunity to have equity-like return with less market risk. However in practice such portfolios have complex design as minimum variance engine needs portfolio constraints to put the concept at work. Investors face today a broad offer of minimum variance methodologies, so how can one compare the different implementations and evaluate what constraints are really needed in minimum variance portfolio? We show in this paper how to read the constraint map and ensure that the necessary is made to make minimum variance engine work. We also list extra constraints that one encounters in existing minimum variance methodologies and discuss how they impact the ability of the strategy to reduce volatility and improve diversification.

Introduction

In the aftermath of the global financial crisis efficient use of risk budgets became a priority for many investors. Minimum variance investing, that attracted a great deal of attention in this regard, allows achieving risk reduction via selecting stocks with high diversification potential: low volatility and/or low correlations to other stocks. Minimum variance uses the principle of portfolio optimization introduced by Harry Markowitz and is a fully invested portfolio with minimal possible volatility. However the sound theoretical principles behind minimum variance approach are not enough to build a real-life minimum variance portfolio. Optimization procedure is sensitive to estimation errors in stock volatilities and correlations. Numerous solutions to the problem exist, including cleaning of covariance matrix with advanced statistical methods and portfolio resampling techniques. But the most widely used solution was and remains the use of portfolio constraints that allow redressing the biases in the optimized portfolio. Hence, minimum variance, as any other optimized portfolio, can benefit from a wisely devised constraint set. But what constraints should be implemented? It happens that different providers of minimum variance solutions have no consensus on the subject: existing minimum variance methodologies have different design, and as a result quite a few differences in the risk and performance of the respective minimum variance portfolios. This article provides an overview of the minimum variance investing approach that is a part of a wider trend called alternative beta. We attempt to give investors tools to interpret the constraints in the con text of minimum variance portfolio and compare different minimum variance methodologies.

Alternative beta story

Why should investors be interested in strategies like minimum variance? Passive investment industry is currently dominated by market-capitalization weighted portfolios, called simply market portfolios or traditional benchmarks. Their popularity amongst investors is to a great extent due to the status of efficient portfolio that the market allocation has in the Capital Asset Pricing Model (CAPM), the classical financial pricing theory developed in 1960s. Today investment industry remains faithful to the mark et portfolio paradigm notwithstanding the important advances in financial theory since the CAPM. One of the crucial findings of academic research since 1970s is the empirical inefficiency of market portfolio. From a theoretical standpoint the attack on the CAPM came first through unrealistic nature of some of its assumptions. Real markets can hardly be approximated by a homogeneous group of investors with similar views and no investor-specific constraints, and an entire field of behavioral finance developed around irrationality and biases of market participants. Another weak point of CAPM turned out to be the replication of the true market portfolio. Indeed, CAPM advocates efficiency of a global market portfolio, aggregating all the possible risky holdings investors can have. In reality, investors can access only market proxies, often in the form of narrow regional indices. Finally, important pricing anomalies were discovered in the 1990s, following the work of Fama and French (1992). It is widely admitted nowadays that risk factors other than the market affect stock returns: value, size, momentum, and later on, volatility. Following these theoretical advances, a family of new approaches to efficient passive investing was developed, known under the name of alternative beta. Alternative beta portfolios address inefficiencies of mar-

ket capitalization-weighted benchmarks: the strategies employ systematic rules in portfolio construction aiming at adding to the broad market beta an extra (risk- adjusted) benefit. It is common to distinguish several subgroups within the alternative beta family: risk-based approaches incorporating risk management objectives in portfolio construction (such as minimum variance and risk parity), fundamental-based strategies built upon return forecasts based on micro or macroeconomic data (such as Fundamental index and weighting by GDP), and diversification-based approaches that ignore completely return as well as risk forecasts and concentrate on smoothing the distribution of portfolio weights (such as equal weight approach). Alternative beta strategies and minimum variance in particular, are meant to be broad portfolios and are not dedicated to a narrow investment theme or a specific sector. This distinguishes them from style portfolios, like value and size, as well as from narrow sector benchmarks. While the concepts behind the alternative beta strategies are widely discussed, the constraints that play an important role in definition of the alternative beta portfolios are often left out. However it is due to constraints that the strategies can be implemented in a systematic way, remain diversified notwithstanding unfavorable market conditions and mitigate risks that are not controlled by the investment model, such as liquidity risk and estimation risk. Without the constraints one is forced to introduce discretionary human oversight, and thus foregos a great deal of transparency and investment discipline that is a clear advantage of passive investing.

Minimum variance portfolio: the basic setup

Technically, to construct a minimum-variance portfolio one needs a forecast of covariance matrix and an optimization engine. The latter is a mathematical procedure that decides which stocks to pick and what weights to give them to obtain the lowest possible volatility of the overall portfolio. As the smallest possible risk is an exact zero, one needs also a constraint that forces the allocation to be non-empty, such as the budget constraint forcing the sum of weights to be equal to one. Further on, to have a broad investment benchmark one would like to avoid having negative weights in the minimum variance allocation. This motivates the second necessary constraint: no short sales.

Diversification constraints

The basic configuration: a covariance matrix, an optimizer and the two constraints above, give an essential minimum variance portfolio. This is not enough however for an investment benchmark for two reasons. First, common sense as well as the financial regulating authorities suggest that a financial index should satisfy a set of diversification criteria. A basic minimum variance portfolio has no control over diversification and is too concentrated to be an eligible investment benchmark. Moreover, an extreme concentration in minimum variance allocation is often a consequence of underestimation of expected covariance, so some stocks seem too attractive to the minimum variance algorithm. Here diversification constraints bring additional benefit of reducing misallocations due to the estimation noise. There exist two main types of diversification constraints: limiting maximal weight per stock or targeting diversification in terms

of effective number of assets, that is the exact number of assets required to construct an equally-weighted portfolio with the same concentration[1] . The constraints in effective number of assets (that are quadratic in portfolio weights), have several advantages with respect to the maximal weight limits (that are linear in weights). In particular, the former allow constructing more diversified portfolios for a given level of maximal weight, as well as preserving the information on the relative riskiness of stocks that is usually erased by conservatively low weight limits. Here we give an illustration of the diversification achieved by both methods. On Figure 1 we show the level of diversification of minimum variance portfolios constructed using the two different types of diversification constraints.

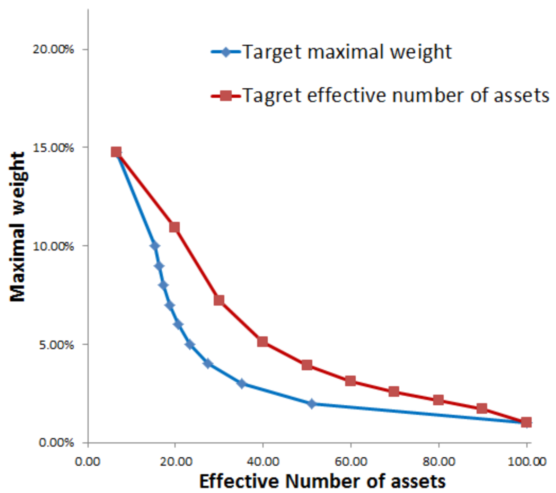

Figure 1: Maximal weight limit versus targeting effective number of assets. Source: Datastream

The portfolios on the blue line were optimized using the linear constraint with the maximal weight threshold gradually shifte d from 1% to 15%. The portfolios on the red line were constructed via optimization with the quadratic diversification constraint targeting effective number of assets ranging from 10 to 100. For any given level of maximal weight the red portfolios are more diversified than the blue ones. For example, if one targets the maximal weight of 5%, the hard limit of 5% will result in a portfolio of 20 effective assets, while the same maximal weight is respected by a red portfolio that is two times more diversified.

Liquidity Risk

The second reason why one needs more sophisticated minimum variance portfolio construction than the basic setup is the control over invisible liquidity risk. This risk is not measured by stocks volatility, and thus the optimized portfolio might allocate large positions to rather illiquid stocks. This can be addressed by adding explicit constraints in terms of stocks absolute or relative liquidity. For example, one can set a threshold of minimum average daily volume that future constituents of minimum variance portfolio should exhibit. Relative thresholds however work better (e.g. selecting N most liquid stocks from the underlying universe), as this keeps portfolio construction free from predefined levels expressed in currency units and suits to a portfolio of arbitrary size. The liquidity screening is better being done before the launch of the optimization process, and not by fine-tuning the optimal allocation eliminating illiquid stocks. The reason is simple: posterior adjustment of minimum variance composition makes it suboptimal, and there will be always another portfolio with the same level of liquidity that will give smaller volatility than the one of the readjusted portfolio. As an alternative to liquidity screening, some providers impose limits on portfolio turnover. However this addresses only partially the liquidity risk. Turnover constraint helps to reduce the risk of treating too big orders in illiquid stocks at once; however

it does not prevent the strategy from gradually accumulating important positions in these stocks. This constraint also introduces a path dependency, so that optimization today depends on the portfolio composition yesterday and further back into the past.

Summary of necessary constraints

An investor willing to allocate to minimum variance and get market beta return with reduced market risk, would be interested in the following necessary set of conditions to get a robust portfolio:

- full investment, or budget constraint
- no short sales
- diversification constraint
- liquidity constraint

Minimum variance portfolio with necessary constraints will be somewhat more volatile than an equivalent portfolio with, for example, only a budget constraint. Each additional constraint forces the solution to depart from the true volatility minimum, at least ex-ante. However, this volatility increase is a price one pays for a robust and controlled investment process that can be implemented in a systematic way and result in a diversified and investable minimum variance benchmark.

Minimum variance portfolio: extra constraints

Once the necessary setup is respected, one can add few other constraints to pursuit additional investment objectives. Two words of caution here: first, any additional constraint raises the portfolio ex-post volatility, as it drives the allocation further away from the true volatility minimum; secondly, it is worth checking if extra constraints are not in conflict with one of the necessary constraints imposed, or with the main objective of the optimization: volatility reduction.

Sector and country constraints

The most common extra constraint is limiting sector and/or country exposure. This constraint actually can be considered as one of the necessary ones if implemented in an objective way. Limiting important factor exposures allows th e portfolio being more representative of the broad market beta and limits the risk of becoming a single-sector bet. However, this risk is limited if the thresholds for sector or country exposures are not variables out of the strategy control. An example is limiting sector allocations to be within a certain range around the current sector exposure of the market capitalization-weighted portfolio. In this case there is no guarantee that exposure to a single sector will not peak following a market bubble.

Individual security capping relative to its market capitalization

This type of constraints is used in the first place to approach minimum variance portfolio to the market capitalization bench mark. A clear secondary objective here is to reduce the tracking error of minimum variance portfolio vis--vis the market portfolio. An example of this constraint is limiting a stock weight to be not more than twice of its weight in the market cap portfolio. Indirectly this constraint also aligns the sector and country exposures of minimum variance portfolio to that of the market portfolio, and addresses to some extent liq-

uidity concerns.

Impact of constraints on portfolio volatility

To estimate the impact of the extra constraints on volatility of the minimum variance portfolio, we selected four commercially available minimum variance indices that are constructed from European stocks and based on different constraint sets. The indices are:

- STOXX Minimum Variance Unconstrained and Constrained Indices (SAXPUNP and SAXPMVP)
- iSTOXX Europe Minimum Variance Index (ISEMVP)[2]
- MSCI Europe Minimum Volatility Index (MXEUMVOL)

The Unconstrained minimum variance index of STOXX (SAXPUNP) has the simplest construction and can be viewed as a proxy of the necessary constrained setup without explicit liquidity constraint. The index includes budget, no short sales and diversification constraints, along with a limit on rebalancing turnover of 5%. The next in terms of constraint complexity is the iSTOXX Europe Minimum Variance index (ISEMVP) that includes necessary constraints with a relative liquidity filter, as well as one extra constraint: limiting the weight of each sector to a maximum of 20% of the portfolio. Further on, the STOXX Europe 600 Minimum Variance Constrained index (SAXPMVP) adds sector, country and factor constraints relative to the STOXX Europe 600 market capitalization-weighted index. And finally, the most complex of the four, the MSCI Europe Minimum Variance index (MXEUMVOL) adds an additional constraint on individual security capping vis--vis its market cap counterpart[3].

Using the backtests of the four indices from July 2002 to February 2013, we estimate their volatilities during the period. All the four indices achieved significant volatility reduction with respect to the European market portfolio (the STOXX Europe 600 Index), as shown on Figure 2:

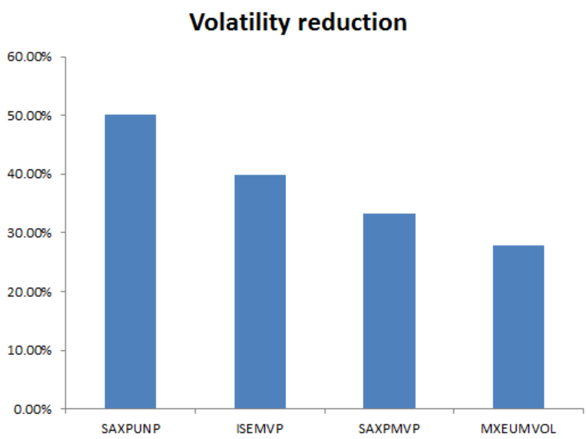

Figure 2: Impact of constraints on volatility of minimum variance indices. Source: Bloomberg, calculation by Ossiam

As one expects, the larger is the set of constraints, the smaller is the volatility reduction achieved by minimum variance portfolios. The less constrained SAXPUNP index has volatility two times smaller than the market index, while MXEUMVOL, the most constrained one, achieves only 25% volatility reduction during the period. In terms of absolute volatility, we summarize on Figure 3 the impact of a growing pile of constraints by plotting the cumulative volatility differences among the indices. SAXPUNP, that is our proxy of the setup with necessary constraints, has annualized volatility of 10.48%. The addition of absolute sector constraint raises the volatility of ISEMVP to 12.64%. Adding further sector, country and factor constraints relative to the market portfolio brings the volatility up to 14.04%

for SAXPMVP. And, finally MXEUMVOL has the highest volatility of 15.18% due to extra relative constraints on individual security weights.

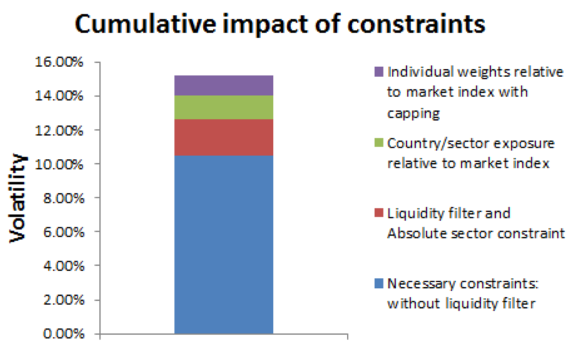

Figure 3: Cumulative impact of constraints on volatility of minimum variance indices. Source: Bloomberg, calculation by Ossiam

Conclusion

Because of the challenges in implementing the minimum variance approach the methodologies differ significantly from one provider to another. Minimum variance portfolios can be very basic, including only full-investment and perhaps long-only constraints: one encounters such "plain-vanilla" minimum variance portfolios in academic research papers. Too simplistic implementation suffers from highly concentrated portfolios and has no control over portfolio liquidity that is unacceptable for an investment benchmark. On the other hand, a heavy set of constraints, especially those binding the portfolio weights to resemble the market capitalizations, might conflict with the very objective of portfolio variance minimization. We showed in this paper that there is a necessary set of constraints; otherwise the minimum variance portfolio is empty or dangerously concentrated and illiquid. Adding constraints beyond the necessary set helps to define extra investment objectives but should be carefully balanced to avoid the conflict with respect to the primary objective of volatility reduction. Next time you face a new minimum variance methodology: check if the necessary constraints are incorporated, then question the role the extra constraints involved.

Notes

[1] This constrained is called also Herfindahl constraint, the name that refers to the Herfindahl-Hirschman index used by antimonopoly regulators to measure concentration inside industrial sectors.

[2] Disclaimer: Ossiam licenses the iSTOXX Europe Minimum Variance Index for the use in exchange-traded fund.

[3] This description represents an approximate comparison of the four indices, since the constraints are not applied in the same way. No index can be strictly considered as a special case of another index with less or more constraints. For example, only the index iSTOXX uses the explicit liquidity constraint while the other three indices rather use the turnover constraint. Besides, there are differences in the implementation of diversification constraints among the indices that we do not detail here.

About Ossiam

Ossiam is a research-driven French asset management firm (authorized by the Autorité des Marchés Financiers) and specializes in delivering smart beta* solutions. Efficient indexing is at the core of Ossiam's business model. The firm was founded in response to a post-subprime crisis demand from investors for simplicity, liquidity and transparency. Given the environment, there was a growing need among investors for enhanced beta exposure and risk hedging. Ossiam is focused on the development of innovative investment solutions for investors via a new generation of indices.

**'Smart beta' refers to systematically managed, non-market-cap-weighted strategies covering any asset class.*

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